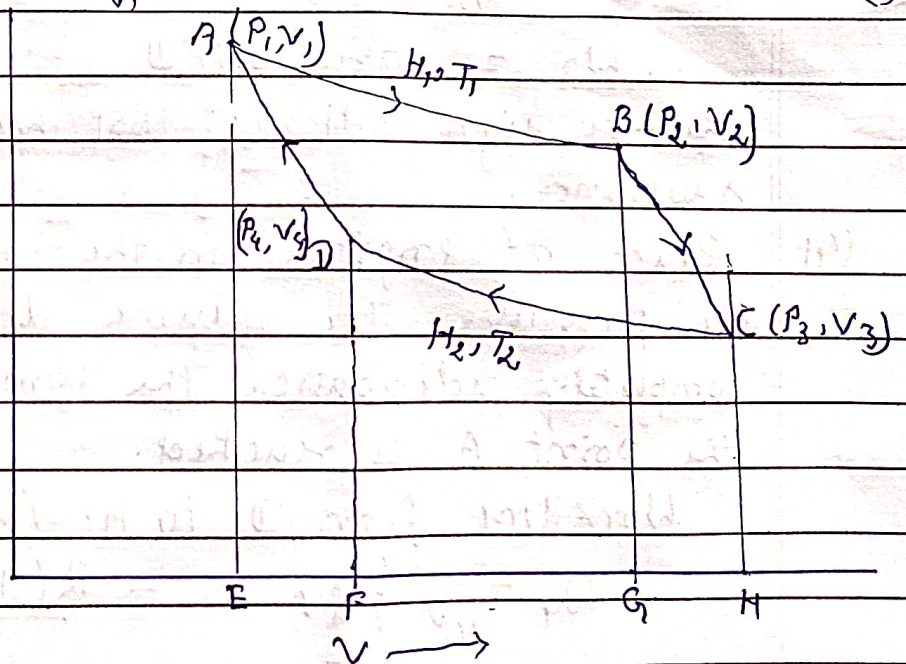


* Carnot's Cycle :-

(1) Place the engine containing the working substance over the source at temp^r. T_1 . The working substance is also at a temp^r. T_1 . Its pressure is P_1 and volume is V_1 as shown by the point A. Decrease the pressure. The volume of the working substance increases. Work is done by the working substance. As the bottom is perfectly conducting to the source at temp^r. T_1 , it absorbs heat. The process is completely isothermal. The temp^r. remains constant. Let the amount of heat absorbed by working substance be H_1 at the temp^r. T_1 . The point B is obtained.

Let us suppose one gram molecule of the working substance
Work done A to B, $W_1 = \int_{V_1}^{V_2} P \cdot dV = RT_1 \log \frac{V_2}{V_1} = \text{Area ABGE}$ (i)

(2) Place the engine on the stand having an insulated top. Decrease the pressure on the working substance. P
The volume increases.
The process is completely adiabatic. Work is done by the working substance at the cost of



its internal energy. The temp^r. falls. The working substance undergoes adiabatic change from B to C. At C, the temp^r. is T_2 .
Work done from B to C (adiabatic process)

$$W_2 = \int_{V_2}^{V_3} P \cdot dV$$

$$= \int_{V_2}^{V_3} \frac{dV}{V^\gamma}$$

But $PV^\gamma = \text{constant} = K$

$$P_2 V_2 = RT_1$$

$$P_3 V_3 = RT_2$$

$$P_3 V_3^\gamma = P_2 V_2^\gamma = K$$

$$\therefore W_2 = \frac{kV_3^{1-\gamma} - kV_2^{1-\gamma}}{1-\gamma} = \frac{P_3V_3 - P_2V_2}{1-\gamma}$$

$$= \frac{R[T_2 - T_1]}{1-\gamma}$$

$$= \frac{R[T_1 - T_2]}{\gamma - 1}$$

$\therefore W_2 = \text{Area } BCHG$ — (ii)

(3) Place of engine on the sink at temp^r. T_2 . Increase the pressure. The work is done on the working substance. As the base is conducting to the sink, the process is isothermal. A quantity of heat H_2 is rejected to the sink at temp^r. T_2 . Finally the point D is reached.

Work done from C to D (isothermal process)

$$W_3 = \int_{V_3}^{V_4} p dV = RT_2 \log \frac{V_4}{V_3} = -RT_2 \log \frac{V_3}{V_4}$$

$\therefore W_3 = \text{area } CHFD$ — (iii)

The -ve sign indicates that work is done on the working substance.

(4) Place of engine on the insulating stand. Increase the pressure. The volume decreases. The process is completely adiabatic. The temp^r. rises and finally the point A is reached.

Work done from D to A. (adiabatic process)

$$W_4 = \int_{V_4}^V p dV = - \frac{R(T_1 - T_2)}{\gamma - 1}$$

$\therefore W_4 = \text{Area } DFBA$ — (iv)

W_2 and W_4 are equal and opposite and cancel each other.

The net work done by the working substance in one complete cycle.

$$= \text{Area } ABGB + \text{Area } BCHG - \text{Area } CHFD - \text{Area } DFBA$$

$$= \text{Area } ABCD$$

The net amount of heat absorbed by the working substance
 $= H_1 - H_2$

Net work = $W_1 + W_2 + W_3 + W_4$

$$W = RT_1 \log \frac{V_2}{V_1} + \frac{R(T_1 - T_2)}{\gamma - 1} - RT_2 \log \frac{V_3}{V_4} - \frac{R(T_1 - T_2)}{\gamma - 1}$$

$$\therefore W = RT_1 \log \frac{V_2}{V_1} - RT_2 \log \frac{V_3}{V_4} \quad \text{--- (v)}$$

The points A and D are on the same adiabatic

$$T_1 V_1^{\gamma - 1} = T_2 V_4^{\gamma - 1}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_4} \right)^{\gamma - 1} \quad \text{--- (vi)}$$

The points B and C are on the same adiabatic.

$$T_1 V_2^{\gamma - 1} = T_2 V_3^{\gamma - 1}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_2}{V_3} \right)^{\gamma - 1} \quad \text{--- (vii)}$$

from eqn. (vi) & (vii), we get,

$$\left(\frac{V_1}{V_4} \right)^{\gamma - 1} = \left(\frac{V_2}{V_3} \right)^{\gamma - 1}$$

$$\frac{V_1}{V_4} = \frac{V_2}{V_3}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

from eqn (v)

$$W = RT_1 \log \frac{V_2}{V_1} - RT_2 \log \frac{V_3}{V_4}$$

$$\therefore W = R \left[\log \frac{V_2}{V_1} \right] [T_1 - T_2]$$

$$\therefore W = H_1 - H_2$$

Efficiency: $\eta = \frac{\text{Useful Output}}{\text{Input}} = \frac{W}{H_1}$

Heat is supplied from the source from A to B only.

$$H_1 = RT_1 \log \frac{V_2}{V_1}$$

$$\therefore \eta = \frac{W}{H_1} = \frac{H_1 - H_2}{H_1}$$

$$\therefore \eta = \frac{R(T_1 - T_2) \log \frac{V_2}{V_1}}{RT_1 \log \frac{V_2}{V_1}}$$

$$\text{or, } \eta = 1 - \frac{H_2}{H_1}$$

$$\therefore \eta = 1 - \frac{T_2}{T_1} \quad \text{--- (viii)}$$

The Carnot's engine is perfectly reversible. It can be operated in the reverse direction also. Then it works as a refrigerator. The heat H_2 is taken from the sink and external work is done on the working substance and Heat H_1 is given to the source at the higher temperature.

The isothermal process will take place only when the piston moves very slowly to give enough time for the heat transfer to take place. The adiabatic process will take place when the piston moves extremely fast to avoid heat transfer. Any practical engine cannot satisfy these conditions.

All practical engines have an efficiency less than the Carnot's engine.